A NEW METHOD FOR THE MODAL ANALYSIS OF OPTICAL FIBERS HAVING SYMMETRICALLY DISTRIBUTED MULTIPLE CORES

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ABSTRACT

A new modal analysis method based on the combination of a point-matching method and a group theoretic approach is proposed by which optical fibers having symmetrically distributed multiple cores on a circumference can be studied. The case of sextet cores is treated. The results of numerical calculations and microwave model experiments are compared.

INTRODUCTION

The recent development of optical fiber fabrication techniques enables one to make a wide range of fiber structures, namely, various distributions of refractive index and various types of boundary geometry.

We have been studying propagation problems of optical fibers belonging to the latter class (1)(2)(3), and have found that the point-matching method (4) was very effective in processing complicated boundary conditions approximately. Consequently, the propagation constants of such optical fibers have been obtained within a reasonable computer time.

Even with this point-matching method, however, boundary conditions could become untractable in the problems of multiple cores when the number of cores is increased.

In this paper, we propose a new method of analysis based on the point-matching method combined with a group-theoretic approach (5) for the study of optical fibers having symmetrically distributed multiple cores on a circumference. The reduction of the number of matching points is possible with this method, and therefore, the analysis of various fiber structures become feasible.

OPTICAL FIBERS HAVING MULTIPLE CORES

Fig.1 shows a typical fiber structure to be treated in this paper. It has symmetrically distributed sextet (step index type) cores on a circumference. We begin with the following pair of wave equations concerning the z-component of electric field E_z and magnetic field H_z in the cores and cladding:

$$(\nabla_t^2 + k^2 - \beta^2)\begin{Bmatrix} E_z \\ H_z \end{Bmatrix} = 0$$

where ∇_t^2 is the Laplacian operator defined in the cross-section, k is the wave number specified in each medium, and β is the propagation constant.

A system of circular cylindrical coordinates as shown in Fig.1 is adopted here to express these fields in terms of Bessel and sinusoidal functions. The field functions in the cladding are written as the linear combination of fields originated from each core.

Boundary conditions for this structure, namely, the continuation conditions of tangential field components on the surface of each core, must be imposed to these field expressions.

POINT-MATCHING METHOD

There is no direct way to solve the above boundary-value problem. To make the problem tractable, the boundary conditions are approximately satisfied at a finite number of points on the boundary surface. A proper choice of these matching points makes a set of homogeneous linear equations on the coefficients appearing in the above field expressions. The determinant of these linear equations should vanish to obtain non-trivial solutions. The roots of this determinantal equation give the numerical values of the propagation constant (1).

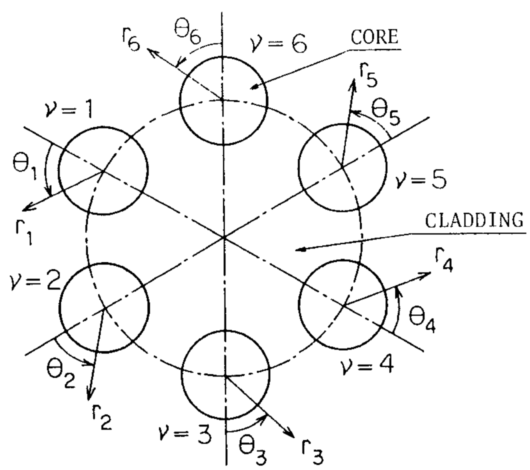

Fig.1 Multiple circular cylindrical coordinates are used for these sextet cores.

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GROUP-THEORETIC APPROACH

The number of the matching points is limited by the memory capacity of a computer to be employed. The use of structural symmetry is an effective way to reduce the number of necessary points. The group theory helps us to treat such symmetry systematically. McIsaac (5) discussed a group-theoretic method for the mode classification of symmetrical-waveguide problems as a general theory. This is, however, not a tool to satisfy complicated boundary conditions. It is also noted that vector wave equations are involved in our electromagnetic wave problems but scalar wave equations are solved in usual quantum mechanical problems although the group theory is used in both cases. We found that the best way to solve our optical fiber problems was the point-matching method combined with such group-theoretic approach.

The outline steps of this approach are as follows:

(1) Use the C_{nv} group defined in the group theory to express the symmetry of our fiber structure,
(2) Derive irreducible representation matrices for the C_{nv} group,
(3) Obtain projection operators from the representation matrices,
(4) Determine the minimum area of the fiber cross-section to be treated,
(5) Select appropriate coordinate systems for using the projection operators, and project the trial field functions defined in the minimum area to the total system.

Then, the field functions have become the symmetry adopted linear combinations.

CASE OF SEXTET CORES

We treated the example case of sextet cores by this new method. The computation time of one determinant value was less than 2 seconds on a computer. The number of necessary matching points was less than 7. The convergence of the value of β/k_c is as shown in Fig.2. Some computed values of β/k_c for normalized frequencies are also shown in Fig.3.

MICROWAVE MODEL EXPERIMENTS

Microwave model experiments to prove the theory were carried out by using sextet Rexolite

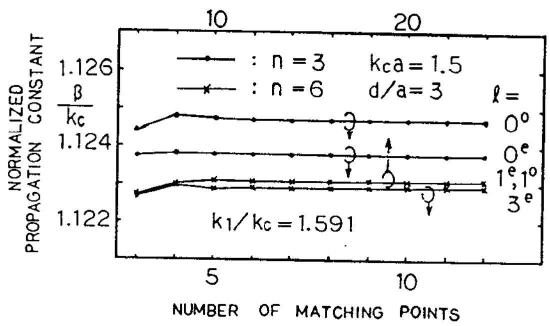

Fig.2 The numerical convergence of propagation constants.

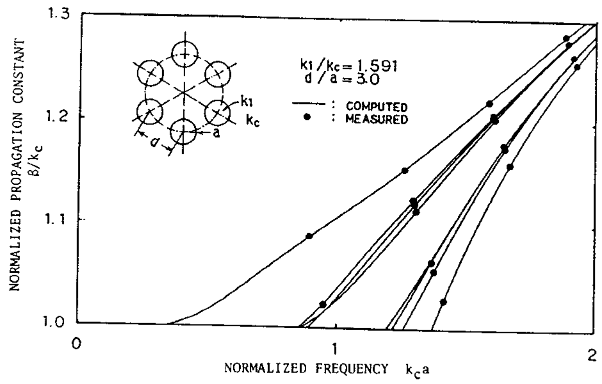

Fig.3 The comparison of the computed propagation constant with measured values.

rods at 2-18 GHz. The Rexolite rods were placed between two conductor plates to form a resonator. The resonance frequency and corresponding guide wavelength were measured to obtain β/k_c. The measured and computed values of β/k_c are compared in Fig.3, which indicates good agreement.

CONCLUSION

The point-matching method combined with a group-theoretic approach is powerful in analyzing the transmission characteristics of a class of optical fiber structures. This method can also be applied to the analysis of microwave dielectric waveguide circuits.

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